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Strength Analysis of Legs of Self Elevating Drilling Units in Transit Conditions

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ABSTRACT

The legs of self elevating drilling units, during transit moves, are to withstand acceleration, drag and gravity bending moments resulting from the motions in the most severe environmental conditions, together with wind moments caused by the most severe storm. For calculations of these bending moments it has to be made a global and local load analysis of the legs in the transit conditions. The leg actual bending moments are to be less than the allowable values calculated taking in account the leg structural configuration, material characteristics and supporting mode. A calculation method of these allowable bending moments for legs of self elevating drilling units, in transit condition, is established based on ABS Rules .

1. Introduction

The self elevating drilling units are complex offshore constructions, which are designed for the most unfavorable of the following loads:

a) Functional loads;
b) Environmental loads;
c) Accidental loads.

during transit conditions and elevated conditions.

In transit conditions, the legs are partly or fully elevated and supported as cantilevers in the hull. Any rolling or pitching motion and transit move, in combination with wind, induce large bending moments in legs and large reaction forces in leg guides, jacking machinery, fixation system, jackhouse and supporting hull structure. For strength analysis, it may be assumed that the stiffness of the hull structure is infinitely higher compared to stiffness of the legs.

In general, the legs are to be designed for static forces, hydrodynamic forces and inertia forces resulting from motions in the most sever environmental transit condition, combined with wind forces resulting from the maximum wind velocity. Wave motions may be obtained either from mode tests or from computations.

Alternately, according to ABS Rules for building and classing of mobile offshore drilling units, the following loads may be considered:

a) in Field Transit Conditions
   - Inertia forces corresponding to a 6° single amplitude of roll or pitch motion at the natural period of the unit;
   - 120% of the gravity moment caused by the legs' angle of inclination;

b) In Severe Storm Transit Conditions
   - Inertia forces corresponding to a 15° single amplitude of roll or pitch motion at the 10 second period of the unit;
   - 120% of the gravity moment caused by the legs' angle of inclination;
   - Wind forces corresponding to a velocity of not less than 51.5 m/s (100 kn).

2. Global load analysis for the transit conditions

For calculation of leg forces (see fig. 1) in transit conditions, when the leg are fully elevated, it assumed that the roll or pitch motion can be described by (the axis of rotation is assumed located in the water plane):

$$\theta = \theta_o \cdot \sin \frac{2 \cdot \pi \cdot t}{T_o} \quad (1)$$

where:

t – time variable;
T_o – natural period of roll or pitch;
θ_o – amplitude of roll or pitch.

The acceleration of a elementary mass located at a distance r from axis of rotation is then:

$$a = -\varepsilon_o \cdot r \cdot \sin \frac{2 \cdot \pi \cdot t}{T_o} \quad (2)$$

where:

$$\varepsilon_o = \left(\frac{2 \cdot \pi}{T_o} \right)^2 \theta_o \quad (3)$$

The maximum forces of elementary mass at a position defined by the coordinate z, are:

a) Transverse forces

- Static force

$$dF_{TS} = 1.2 \cdot g \cdot m(z) \cdot \sin\theta_o \cdot dz \qquad (4)$$

- Inertia force

$$dF_{TD} = m(z) \cdot \varepsilon_o \cdot z \cdot dz \qquad (5)$$

- Wind force

$$dF_W = 1/2\rho C_S(z) \cdot A(z) \cdot v(z)^2 \cdot \cos\theta_o \cdot dz \qquad (6)$$

b) Axial forces

- Static force

$$dF_{LS} = 1.2 \cdot g \cdot m(z) \cdot \cos\theta_o \cdot dz \qquad (7)$$

- Inertia force

$$dF_{LD} = m(z) \cdot \varepsilon_o \cdot d \cdot dz \qquad (8)$$

where:

g – acceleration of gravity;
$m(z)$ – elementary mass of the leg at position z;
$v(z)$ – wind velocity at position z;
ρ – mass density of air;
$C_S(z)$ – shape coefficient at position z;
$A(z)$ – projected wind area of elementary mass at position z.

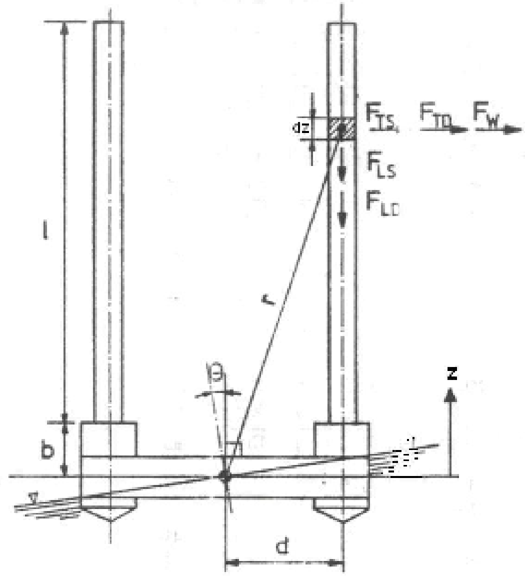

Fig. 1
Motion of a jack-up platform in transit

The leg bending moments, shear force and axial force may be obtained by integration of the load intensities over leg length:

$$F_{TS} = 1.2 \cdot g \cdot \sin\theta_o \cdot \int m(z) \cdot dz \qquad (9)$$

$$F_{TD} = \varepsilon_o \cdot \int m(z) \cdot z \cdot dz \qquad (10)$$

$$F_W = 1/2\rho\cos\theta_o \int C_S(z) \cdot A(z) \cdot v(z)^2 \cdot dz \qquad (11)$$

$$M_{TS} = 1.2 \cdot g \cdot \sin\theta_o \cdot \iint m(z) \cdot dz \cdot dz \qquad (12)$$

$$M_{TD} = \varepsilon_o \cdot \iint m(z) \cdot z \cdot dz \cdot dz \qquad (13)$$

$$M_W = 1/2\rho\cos\theta_o \iint C_S(z) A(z) v(z)^2 dz\, dz \qquad (14)$$

$$F_{LS} = 1.2 \cdot g \cdot \cos\theta_o \cdot \int m(z) \cdot dz \qquad (15)$$

$$F_{LD} = \varepsilon_o \cdot d \cdot \int m(z) \cdot dz \qquad (16)$$

By assuming that the mass, wind area and shape coefficient are uniformly distributed over the length and that the wind profile is defined by the relation:

$$v = v_R \cdot \left(1 + 0.137 \ln\frac{z}{z_o}\right) \qquad (17)$$

where:

v_R – reference wind velocity is defined as wind velocity overaged over one minute (sustained wind) at 10 m above the still water level (see fig. 2);
z_o – reference height (z_o = 10 m);

the following explicit expressions are obtained:

c) Transverse forces

- Static force

$$F_{TS} = 1.2 \cdot g \cdot M_L \cdot \sin\theta_o \qquad (18)$$

acting at : $z_S = l \cdot (1/2 + b/l)$ (19)

- Inertia force

$$F_{TD} = M_L \cdot \varepsilon_o \cdot l \cdot (1/2 + b/l) \qquad (20)$$

acting at:

$$z_D = 2/3 \cdot l \frac{1 + 3 \cdot b/l + 3 \cdot (b/l)^2}{1 + 2 \cdot b/l} \qquad (21)$$

- Wind force

$$F_W = 1/2 \cdot \rho \cdot C_{sL} \cdot A_L \cdot v_R^{\ 2} \cdot z_o \qquad (22)$$

acting at: $z_W = \dfrac{z_R - z_L}{\cos\theta_o} + b$ (23)

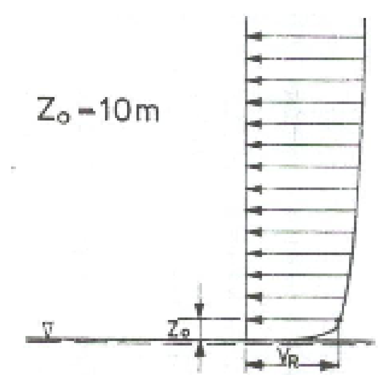

Fig. 2
Wind profile

d) Axial forces
- Static force

$$F_{LS} = 1.2 \cdot g \cdot M_L \cdot \cos\theta_o \quad (24)$$

- Inertia force

$$F_{LD} = M_L \cdot \varepsilon_o \cdot d \quad (25)$$

where:

M_L – total mass of that portion of the leg which is located above upper guides;

A_L – total projected area of that portion of the leg which is located above upper guides;

C_{sL} – average shape coefficient of area A_L;

$$c = 0.85 \cdot \cos\theta_o \cdot [(z_H / z_o)^{1.18} - (z_L / z_o)^{1.18}] \quad (26)$$

$$z_R = 0.85 \cdot z_o \cdot \frac{(z_H / z_o)^{2.18} - (z_L / z_o)^{2.18}}{(z_H / z_o)^{1.18} - (z_L / z_o)^{1.18}} \quad (27)$$

z_H – vertical distance from the still water level to the top of the leg;

z_L – vertical distance from the still water level to lower exposed point of the leg (at upper guides).

The distributions of the combined bending moment and shear force are shown in fig. 3. The most highly loaded part of a jack-up leg is at the upper guide. In principal, the leg bending moment is reacted partly by horizontal forces from the guides and partly by vertical forces from the jacking mechanisms. The relative distribution between horizontal and vertical forces may be defined by a factor β:

$$\beta = \frac{Part\ of\ bend.mom.reacted\ by\ vertical\ forces}{Total\ bending\ moment}$$

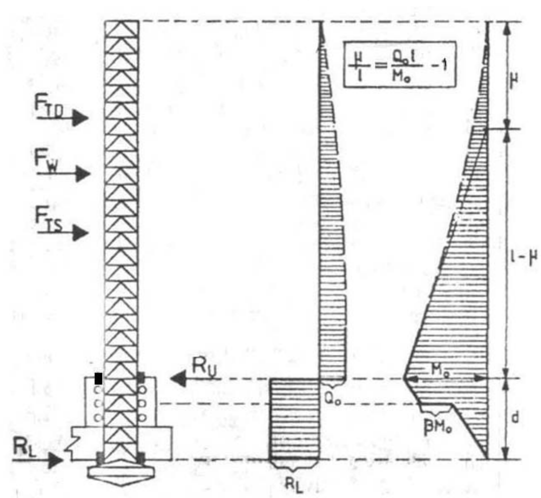

Fig. 3
Bending moment and shear force distribution in transit

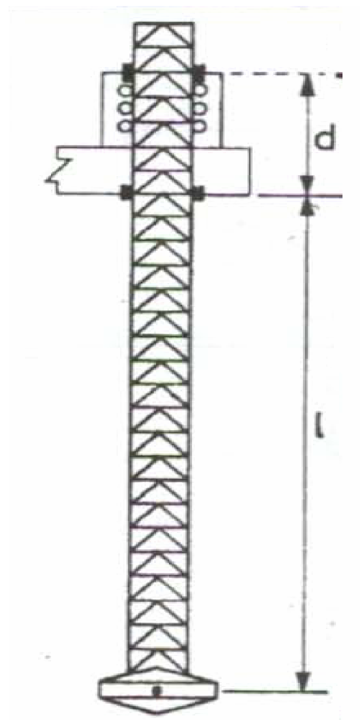

Fig. 4
Partly elevated legs

The horizontal reactions in guides are following:

- Lower guides: $R_L = (1 - \beta) \cdot \frac{M_o}{d}$ (28)
- Upper guides: $R_U = R_L + Q_o$ (29)

where :

$$Q_o = M_o / b_o \quad (30)$$

b_o – lever of global transverse force.

Preliminary calculations show that it may considerer enough exactly:

$$b_o = 2/3 \cdot l \cdot (1 + 2 \cdot b / l) \quad (31)$$

In transit conditions, when the legs are partly elevated (see fig. 4), the part of the legs, sunk into water, is subjected in addition, to the drag forces due to transit speed and due to roll and pitch motion.

The phenomenon is very complex and it can be analyzed similarly as the legs are fully elevated.

3. Local load analysis for the transit conditions

In addition to global shear forces and bending moments over the length of the legs, it is necessary to calculate also the local distribution of forces and bending moments in lattice legs. The horizontal guides reactions in lattice legs (see fig. 5) has to be considered in local analysis. Usually, the analysis of such structures is carried out using computer, considering them as space frame structure or finite element structure.

The bending of the chord member it may obtain by a simple beam analysis of the leg where the guide forces are applied in realistic manner.

For the chord member, the most stressed positions of the guides is at middle of its span (see fig. 6).

For the braces, the most axial stressed positions of the guides is at their joint (see fig. 7).

To calculate the local maximum bending moment in chord member, they may consider a cut segment of chord with length of 5 spans which is analyzed as continuous beam on 6 rigid supports and it may suppose that the guide reaction *R* acts as a concentrated force at midspan of middle member (see fig. 8).

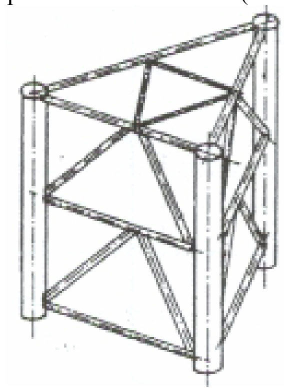

Fig. 5
Lattice leg section

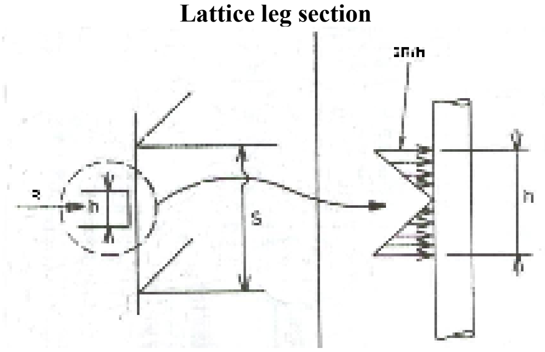

Fig. 6
Action of guides on chord member at middle span

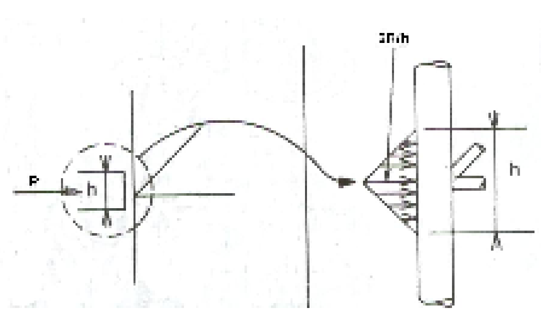

Fig. 7
Action of guides on chord member at joint

The maximum bending in chord is induced by the upper guide reaction R_u and is given by relation:

$$M_l = m_o \cdot R_U \qquad (32)$$

where:

m_o – bending moment in chord induced by a unitary force acting in same point as force R_U;

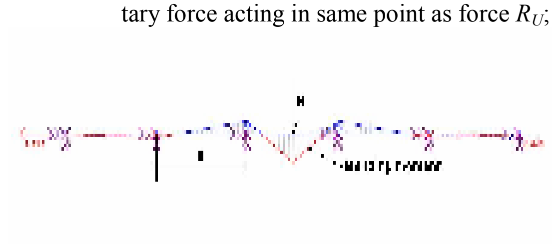

Fig. 8
Idealized chord for local analysis

The braces form more complex structure and it need use space frame analysis or/and finite element analysis by computer.

4. Calculation method of the allowable bending moments of legs

In transit condition, as it is shown above, the most stressed points of the leg chords of the legs are at upper guides, during action of combined external loadings. In these points, leg chords are subjected to axial compression in combination with compression due to global and local bending and to global shear forces so according to ABS Rules for building and classing of mobile offshore drilling units – edition 2008, 3-2-1/3.3 and 3.7.1 and taking in account the characteristics and axial stress level of the legs, the actual stresses are to fulfill the following requirements:

$$f_a / F_a + f_b / F_b \le 1.0 \qquad (33)$$

$$f_s \le F_y / FS_s \qquad (34)$$

where:

f_a – computed axial compressive stress;
f_b – computed compressive stress due to bending;
f_s – computed shear stress;
F_y – specified minimum yield point or yield strength;
F_a – allowable axial compressive stress, which is to be the least of the following:

- Yield stress F_y divided by factor of safety for axial stress $FS_a = 1.25$;
- Overall buckling stress F_{cro} divided by factor of safety specified FS_{cro};
- When $D/t > \dfrac{E}{9 \cdot F_y}$, local buckling stress F_{crl} divided by factor of safety for axial stress $FS_a = 1.25$;

F_b – allowable axial compressive stress due to bending, which is to be the least of the following:

- Yield stress F_y divided by factor of safety for axial stress $FS_a = 1.25$;
- When $D/t > \dfrac{E}{9 \cdot F_y}$, local buckling stress F_{crl} divided by factor of safety for axial stress $FS_a = 1.25$;

$F_{cro} = F_y - (F_y^2 / 4 \cdot \pi^2 \cdot E)(K \cdot s / r)^2$ and

$$FS_a = 1.25 \cdot \left[1 + 0.15 \frac{K \cdot s / r}{\sqrt{2 \cdot \pi^2 \cdot E / F_y}} \right]$$

when $K \cdot s / r < \sqrt{(2 \cdot \pi^2 \cdot E / F_y)}$

$$F_{cro} = \pi^2 \cdot E / (K \cdot s / r)^2$$

$$FS_a = 1.44$$

when $K \cdot s / r \geq \sqrt{(2 \cdot \pi^2 \cdot E / F_y)}$

$$FS_s = 1.88$$

E – modulus of elasticity;
s – unsupported length of chord;
K – effective length factor which accounts for support conditions at ends of length s. For cases where lateral deflection of end supports may exist, K is not to be considered less than 1.0;
r – radius of gyration;
D – mean diameter of cylindrical shell;
t – thickness of cylindrical shell (expressed in the same units as D);

Stresses f_a, f_b and f_s are calculated with relations:

$$f_a = F_o / A_o \quad (35)$$

$$f_b = M_o / W_o + M_l / W_l \quad (36)$$

$$f_s = R_U / A_s \quad (37)$$

where:

$$F_o = F_{LS} + F_{LD} \quad (38)$$

Preliminary calculations indicate that it may cosiderer: $F_o \leq 1.5 \cdot g \cdot M_L$ (39)

A_o – chord shear area ;
M_o – actual maximum combined bending moment at upper guides (see fig. 3);
W_o – equivalent leg section modulus;
M_l – maximum local bending moment in chord at upper guide;
W_l – chord section modulus;
A_s – chord shear area;
R_U – horizontal reaction in the upper guide (see fig. 3).

Taking notice of always the chords of legs are designed as the local and overall buckling strength to be above the yield strength, namely:

$$F_{cro} / FS_{cro} \leq F_y / FS_a$$

$$D/t \leq \frac{E}{9 \cdot F_y}$$

the relation (33) becomes:

$$f_a + f_b \leq F_y / 1.25 \quad (40)$$

By substitutions in (40) the relations (28), (29), (30), (31), (32), (35), (36), (38), (39) and separating M_o, it obtains inequality:

$$M_o \leq \frac{0.8 \cdot F_y - 1.5 \cdot \dfrac{g \cdot M_L}{A_o}}{\dfrac{1}{W_o} + \dfrac{(1-\beta) \cdot m_o}{W_l} \cdot \left(\dfrac{1-\beta}{d} + \dfrac{1}{b_o} \right)} \quad (41)$$

The right member of this inequality is the expression of allowable bending moments of the leg from the compression criteria.

It is dependent only of constructive characteristics of the leg, its material and supporting mode so it is used for all transit conditions.

It should be noticed that the factor β can be usually 0, when jacking system can supports only compressive forces and the resultant of transverse and axial forces of the leg act without of the supporting jacking system area.

In these conditions the relation (41) becomes:

$$M_o \leq \frac{0.8 \cdot F_y - 1.5 \cdot \dfrac{g \cdot M_L}{A_o}}{\dfrac{1}{W_o} + \dfrac{m_o}{W_l} \cdot \left(\dfrac{1}{d} + \dfrac{1}{b_o} \right)} \quad (42)$$

By substitutions in (34) the relations (28), (29), (30), (31), (37) and separating M_o, it obtains inequality:

$$M_o \leq \frac{0.532 \cdot F_y \cdot A_s}{(1-\beta)/d + 1/b_o} \quad (43)$$

The right member of this inequality is the expression of allowable bending moments of the leg from the shear criteria.

If $\beta = 0$, the relation (43) becomes:

$$M_o \leq \frac{0.532 \cdot F_y \cdot A_s}{1/d + 1/b_o} \quad (44)$$

According to ABS Guide for buckling and ultimate strength assessment for offshore structures, Section 5 – edition 2008, the strength of a tubular joint (see fig. 9) subjected to combined axial and bending loads, is to satisfy the following state limit:

$$\frac{|P_D|}{|\eta \cdot P_u|}+\left(\frac{M_{IPB}}{\eta \cdot M_{uIPB}}\right)^2+\frac{|M_{OPB}|}{|\eta \cdot M_{uOPB}|}\leq 1 \quad (45)$$

P_D – axial load in the brace member;
P_u – critical joint axial strength;
M_{IPB} – in-plane bending moment in the brace member;
M_{uIPB} – critical joint bending moment strength for in-plane bending in the brace member;
M_{OPB} – out-of-plane bending moment in the brace member;
M_{uOPB} – critical joint bending moment strength for out-of plane bending in the brace member;
η – maximum allowable strength utilization factor;

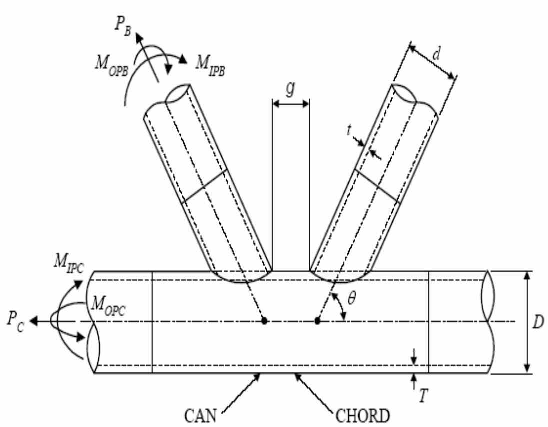

Fig. 9
Geometry of Tubular Joints

When horizontal upper guide act on such joint (see fig. 7), the actual axial forces P_D and bending moments M_{IPB} and M_{OPB} in this joint, are maximum and linear dependent from reaction R_u:

$$P_D = k_D \cdot R_U \quad (46)$$

$$M_{IPB} = k_{IPB} \cdot R_U \quad (47)$$

$$M_{OPB} = k_{OPB} \cdot R_U \quad (48)$$

The coefficients K_D, K_{IPB}, K_{OPB} can be calculated using computer, considering lattice leg with joints as space frame structure or finite element structure.

By substitutions in (45) the relations (28), (29), (30), (31), (46), (47), (48) and separating M_o, it obtains inequality:

$$M_o \leq \Phi\left(\frac{1-\beta}{d}+\frac{1}{b_o}, k_D, k_{IPB}, k_{OPB}, \eta\right) \quad (49)$$

The right member of this inequality is the expression of allowable bending moments of the leg from the joint strength criteria.

If $\beta = 0$, the relation (49) becomes:

$$M_o \leq \Phi\left(\frac{1}{d}+\frac{1}{b_o}, k_D, k_{IPB}, k_{OPB}, \eta\right) \quad (50)$$

The allowable bending moment of the leg is to be fulfill all three criteria: the compression, shear and joint strength criteria, namely inequalities (42), (44) and (50).

Using this method, it was calculated the allowable bending moment of the legs for the self elevating drilling unit PROMETEU. It found the allowable value of 86141 kNm.

5. Conclusions

The paper shows the complexity of the strength analysis of the legs of self elevating drilling units in transit conditions and gives the possibility to develop this analysis deeply.

Also, it shows for authorities, classification societies, designers or shipyards an efficient calculation method of the allowable bending moment of the legs to verify on board, quickly and simply their strength in transit conditions.

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